Ass. 1a) The Alonso-Muth-Mills (AMM) is a model for determining and evaluate rent and prices on land and housing.

This model assumes: Monotonic city where all work is, Supply is fixed (inelastic), demand is elastic ⇒ Demand determines the price/rent. Price and rent is a function of distance, d, to city center. households are rational and identical, housing structure is fixed, and houses are homogeneous, The site of a Real estate is unique, i.e. hetrogeneous. ~~Households will locate where their willingness to pay is fulfilled.~~ Households commute lineary at a commuting cost, k, to the city center where work is. ~~Rent~~ Land is allocated to the user with the highest willingness to pay.

The model: Households budget constraint is given by:

$y = R(d) + kd + x^{\circ}$ (1), where $R(d) :=$ housing rent which is a function of the housevalue times a capitalization rent, i. $y :=$ households income, $kd :=$ commuting cost to city center, $x^{\circ} :=$ other cost $= y - kb - (r^a q + c)$ (2), where $kb :=$ commuting cost to city border, $r^a q :=$ aggricultural rent which is a compensation for the lost alternative cost (e.g. the land could have been used to farming). $c :=$ structure rent which is the capitalized value of building the construction.

(1) can be rewritten with respect to (wrt.) $R(d)$:

$R(d) = y - kd - x^{\circ}$ (3), putting (2) into (3) we can simplify the expression for housing rent:

$R(d) = y - kd - (y - kb - (r^a q + c)) = k(b-d) + r^a q + c$ (4), where $k(b-d) :=$ location rent, which is highest in the city center, and lowest at the city border where $d=b$. The net result of (4) is that housing rent ~~depend~~ consist on location rent, structure rent and aggricultural rent.

Analysing (4) wrt. d: $\frac{\partial R(d)}{\partial d} = -k$ Interpretation:

$-k$ is the marginal change in Rent(housing) as distance increases

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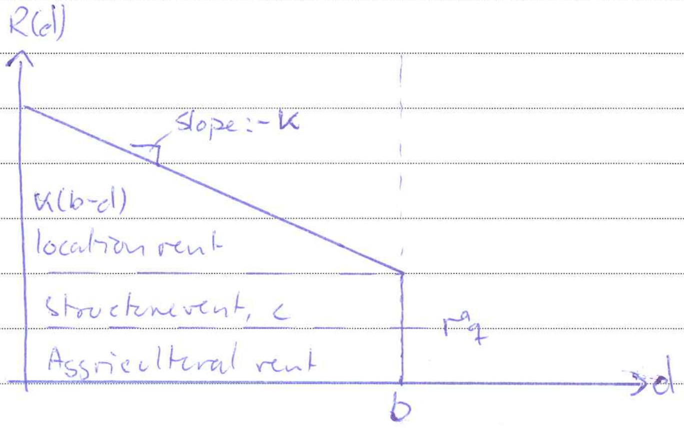
with one unit, keeping all other ~~cons~~ variables constant.

By taking the partial derivative of $R(d)$ wrt. d we get the housing rent gradient i.e. the slope.

b)



As we see from the figure, and as we saw by taking the $\frac{\partial R(d)}{\partial d} = -K < 0$, location rent will fall as we move out of the city center and the distance, d, increases.

Rent in the city center will be given by: $R(0) = K(b-0) + r^a q + c = Kb + r^a q + c$.

Rent ~~at~~ the city border will be given by: $R(d=b) = K(b-b) + r^a q + c = r^a q + c$. Note: at $b=d$, the border, you only pay for structure rent and aggricultural rent, no location rent.

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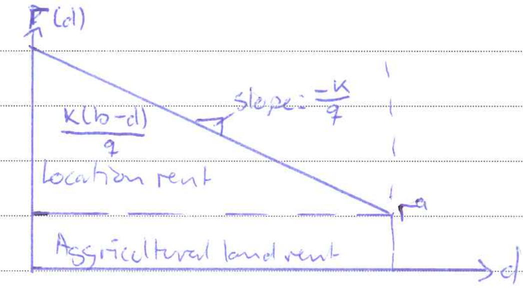
1c) Land rent is ~~given~~ derived by taking (4) and multiply it with $\frac{1}{q}$. this yields the expression for land rent, $r(d)$:

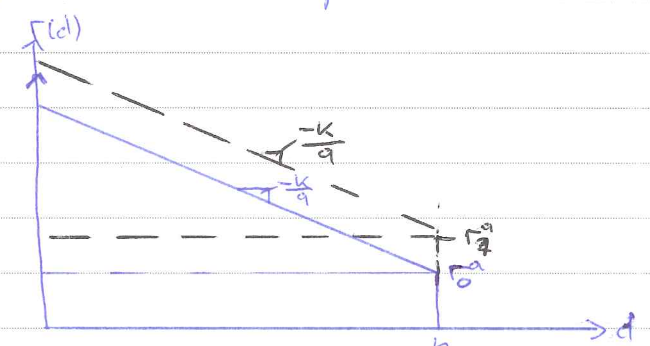
$r(d) = \frac{R(d)}{q} (= \frac{r^a q}{q} + \cancel{\frac{c}{q}} + \frac{k(b-d)}{q} = r^a + \cancel{c} \frac{k(b-d)}{q}$ (5), of corse, structure rent, c will not be a part of this expression (5). Analysis of (5): $\frac{\partial r(d)}{\partial d} = \frac{-k}{q} < 0$. Interpretation: $-\frac{k}{q}$ is the marginal change in land rent as distance increase with one unit. Keeping other variables fixed. Illustration:



1d) In the AMM model, we can put a time index, ~~t~~ t, at $R_t(d)$, $r_t(d)$, $p_t(d)$:= price of land at time t, and $P_t(d)$:= price of Property at time t. This time index comes from b_t which is the only part of the AMM model that considers growth and changes in this model, $b_t = b_0 e^{gt}$, where g := border growth which is the half of population growth (i.e. population growth = 2g).

However it is common to consider a ~~d~~increase in aggricultural rent, a exogenous shock which makes r^a increase to a higher level. This can be explaind by inflation and/or changes in the tecnology.

Illustration of an exogenous shock in r^a which makes r^a increase.



In this illustration, we see the change (increase) in r^a in black and stripped lines, notice that the slope is unchanged, the border is unchanged, but the land rent increases with the exact same

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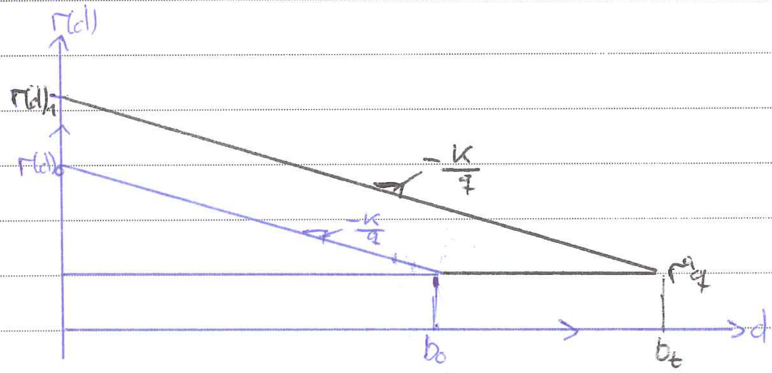
d) cont

amount that aggricultural rent increases with.

Now, assume that g increases with 2% per anno (p.a.), which makes $b_t = b_0 e^{gt}$ increase. An increase in border at time t makes the border shift outwards, which makes the land rent at a given place d increase.

Illustration:



In this illustration, we se that the border has expand from b_0 to b_t, and rent on land has encreased as a result of the expansion from $r(d)_0$ to $r(d)_1$.

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1e) Assuming profit maximizing land owner and that a site is for residential use only. The other assumptions from AMM hold.

Willingness to pay: $P = \alpha + \beta F$, here $\alpha :=$ attributes coefficient which is a given level of a real estates attributes. we assume this to be positive. $\frac{\partial P}{\partial F} = -\beta < 0$. Interpretation:

$-\beta$ is the marginal change in willingness to pay as density increases with one unit keeping all other variables fixed.

Cost of construction: $C = \mu - \tau F$, $\mu :=$ start-up-cost accosiated with construction e.g. contracts, buying land etc. the μ coefficient is assumed to be positive. Assuming $\alpha > \mu$.

$\frac{\partial C}{\partial F} = \tau > 0$. τ is the marginal increase in cost of construction as density increases with one unit keeping other variables fixed.

$\Pi_{BOA} :=$ profit from housing flor area $= P - C = \alpha - \beta F - \mu - \tau F$
$= \alpha - \mu - F(\beta + \tau)$.

$\Pi_{TOA} :=$ profit from land area $= F(P-C) = F\alpha - \beta F^2 - F\mu - \tau F^2$
$= F(\alpha - \mu) - F^2(\beta + \tau)$. To find the optimal level of F for the profit maximizing land owner, we take the derivative of Π_{TOA} wrt. F, sett it equal to zero, and solve for F.

$$\frac{\partial \Pi_{TOA}}{\partial F} = (\alpha - \mu) - 2F(\beta + \tau) = 0$$, solve for F.

$$\alpha - \mu = 2F(\beta + \tau) \quad \Big| \cdot \frac{1}{2(\beta+\tau)}$$

$$\underline{\frac{\alpha - \mu}{2(\beta + \tau)} = F^*}$$ optimal density

By putting F^* into Π_{TOA} and solve for P, we get the optimal price (willingness to pay): $P^* = \frac{(\alpha - \mu)^2}{4(\beta + \tau)}$

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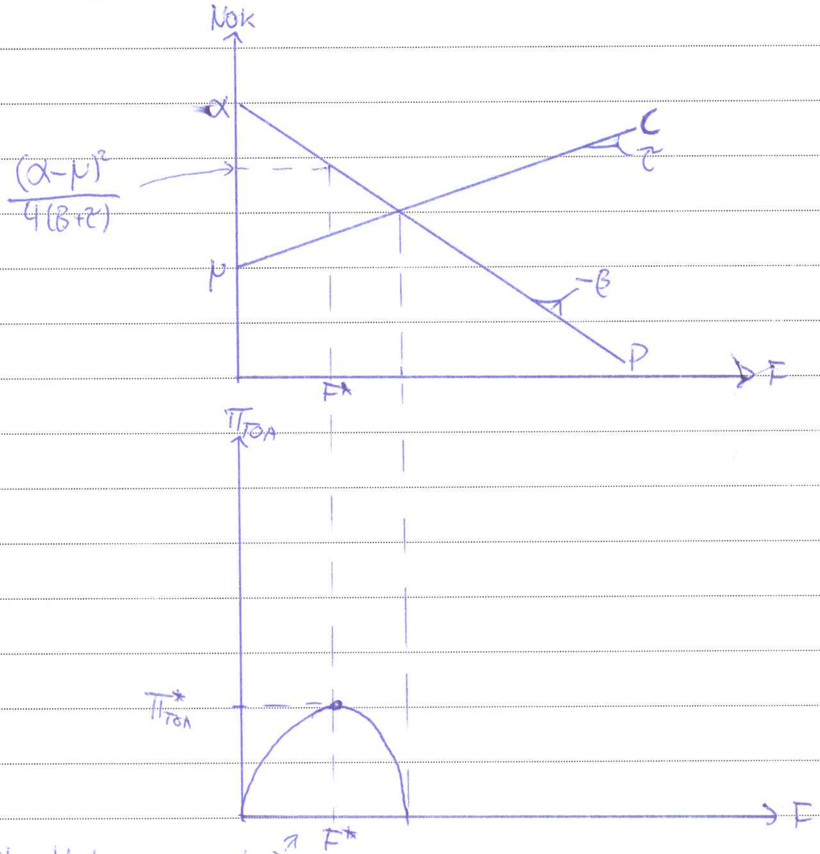
1e) Illustration: (NOK = norske kroner)



Should be symmetric

f) If the attractiveness of the land increases, i.e. α increases, then the function p would parallell move uppwards, because α increases to a higher level of NOK and I assume $-\beta$ remains constant/unchanged. The result of a new P-function would be a higher optimum level of F, because I also assume that C-function remains unchanged. We would get a new optimal price at a higher level of NOK which means the profit would increase.
Illustration at the next page.

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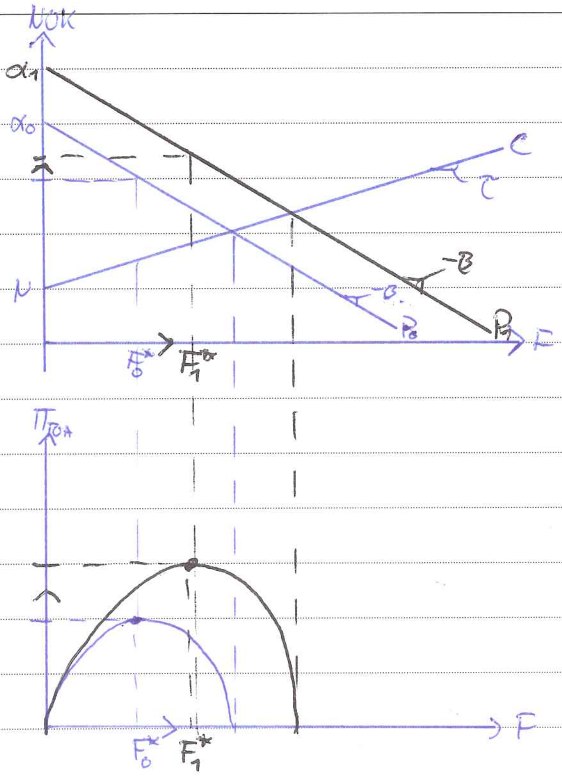
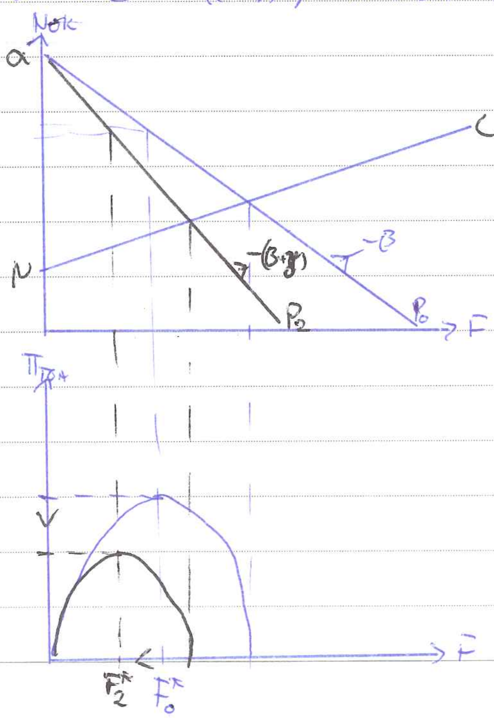


g) Now, P changes, ~~...~~ $P = +\alpha - \beta F - \gamma f$, where

f exogen

$\frac{\partial P}{\partial F} = -\beta - \gamma$: Interpretation: $-\beta - \gamma$ is the marginal change in P due to an increase in f, keeping all other variables fixed. The willingness to pay gradient, i.e. the slope will now be $-(\beta + \gamma)$, which is steeper than $-\beta$.

Illustration.



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g) What happens? As we see on the graph, the P-function pivots, the slope get steeper, $|-(\beta+\gamma)| > |-\beta|$. This make the optimal density, F^*, decrease to F_2^*, and as a result, the profit decreases significantly. Also the price get lower, i.e. the optimal price.

Analytically we get:

$$F_2^* = \frac{\alpha-\mu}{2(\beta+\tau+\gamma)} > 0.$$

$$P_2^* = \frac{(\alpha-\mu)^2}{4(\beta+\tau+\gamma)} > 0.$$

h) External effects is defined as the effect that agent A imposes on agent B, without ~~taking~~ Agent A taking it in to consideration.

This leads often to a suboptimal solution wrt. the ~~society~~ best public solution. Because it lies in human nature that we priorities us self before other, and want to maximize our own utility, without taking into consideration the best public solution. An unregulated market often lead to this.

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Ass. 2a) The Dipasquale-Wheaton (DW) model is ~~a~~ model to determine and analyse the overall effects of economic changes wrt the real estate market. The DW model assumes ~~stock~~ supply is fixed in the short run, demand for space is elastic and does not ~~ta~~ take into consideration who the demander is (objective), and equilibrium is maintained across periods.

The DW model consist of four quadrants which represent four different markets. The northeast corner is the rent determinition ~~in~~ real estate (RE) market, there Rent and stock of space is on the axises, and both of them is depenpendings on m^2 (square meters). Demand is here a function of rent and macroeconomic effects. Demand is assumed to be equal to supply (stock of space pr. m^2).

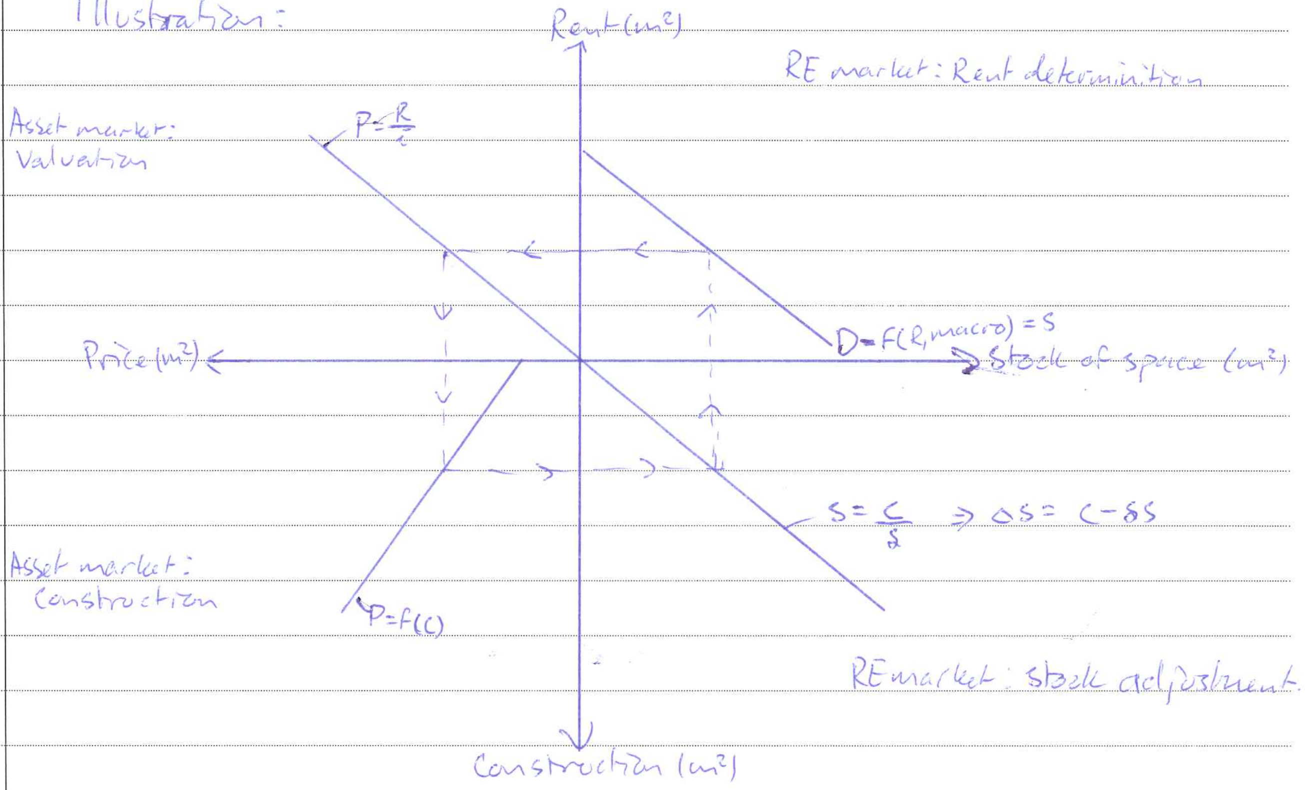
~~In the north~~ The demand function can shift out wards or inwards due to macroeconomic stimuli. Demand curve is falling wrt stock of space (as normal).

In the northwest corner we have Rent and Price on the axis, both a function of m^2. Price is equal to rent divided by a capitalization rate, i, which is a function of tax-regulations, long term interest rate, risk and future growth in rents. The price function can pivot as i (capitalization rate) changes.

In the southwest corner, we have Price and construction on the axis, both a function of m^2. Price is here a function of replacement cost, this is a supply curve. And shows the relationship between construction and price. This function starts at a given price, ~~and~~ due to start-up cost etc. This function can shift sideways due to public regulations.

In the southeast corner, we have construction and stock of space on the axis, both a function of m^2. This is the Real estate market for stock adjustment, Where stock adjustment, $s = \frac{c}{\delta}$, where c := construction of new buildings, and δ := depreciation of existing structures. We can rewrite this as $\Delta s = c - \delta s$, where Δs := change in stock adjustment, and δs := depreciation of existing structures.

Illustration:



This quadrant between the markets show the relationship between Rent determination, valuation, construction and stock adjustment. NB: property markets are here called RE market.

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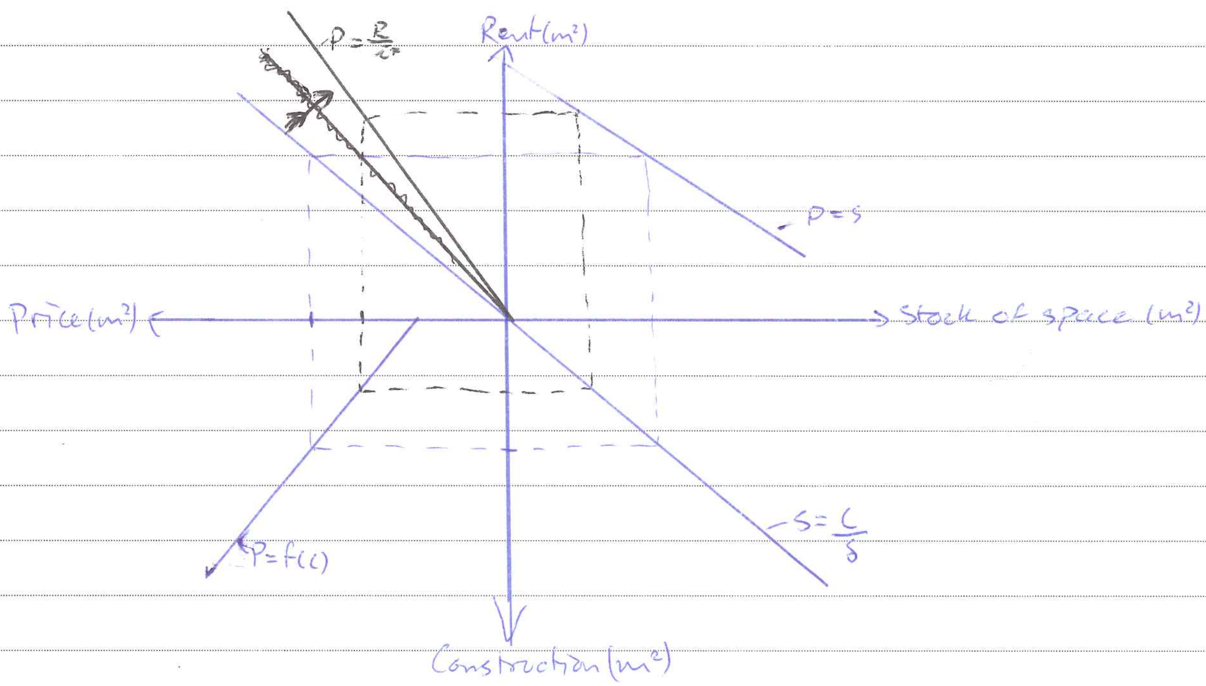
2b) Now, increase in i. An increase in i will make the price function pivot due to $\frac{R}{i}$. The price function will get a steeper slope, because the price will decrease, assuming R is unchanged at the moment of increase in i.

When the price get smaller, we get less construction, less construction leads to a decrease in stock of space pr. m^2. less stock of space leads to people be willing to pay more to get some space, so the rent increases.

Illustration:



A result of this increase in i, is that the quadrant changes figure, with a higer level of rent, less price, construction and stock of space.

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Ass 3. a) and b)

Assume: many places firm can locate, all firms have equal Q, p, A, C, f and identical products, land is allocated to the firm who pays the highest rent. Assume perfect competition and that in the long-run profit $= 0$.

Firms profit function: $\Pi = Q(p-A-sd) - C - r_c(d)f = 0$ (6)
where Q := quantity produced, sd := transportation cost, C := fixed cost, f := lot size, $r_c(d)$:= rent for firm. Solve (6) w.r.t. $r_c(d)$

$$Q(p-A-sd) - C = r_c(d)f \quad \Big| \frac{1}{F}$$

$$\frac{Q(p-A-sd)-C}{F} = r_c(d) \quad \text{: rent for firm.}$$

to see the rent gradient of the firm, we take the derivative of $r_c(d)$ w.r.t. d

$$\frac{\partial r_c(d)}{\partial d} = -\frac{Qs}{F} < 0$$

Interpretation: $-\frac{Qs}{F}$ is the marginal change in $r_c(d)$ as F, lot size increase with one unit, keeping all other variables constant.

Now, let us assume we have office-firms and industry firms, and they are profit maximizers and now Q, and f is not fixed. Such that they can locate where it is optimal due to Q, f and so $j \in \{Office = 0, Industry = I\}$

$$\frac{\partial r_c^j(d)}{\partial d} = -\frac{Q_j s_j}{F_j} < 0.$$ Same interpretation.

Assuming Office firms have high Q w.r.t. f and high transport cost, they will then have a steep slope, a high value on $\left| \frac{-Q_0 s_0}{F_0} \right|$.

Assuming that industry firms need much space such that Q w.r.t. f get small, allso they have lower transportation cost then Office firms.

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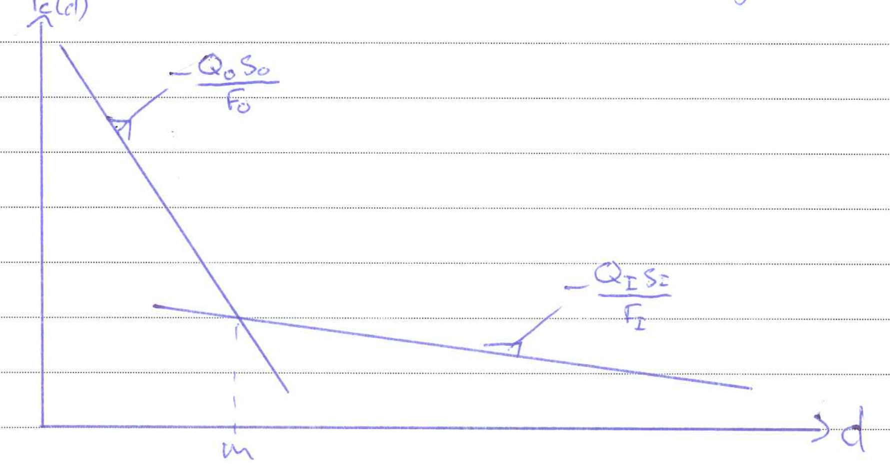
We get this illustration wrt. firms j.



We see that Office firms will locate close to the city center, where the rent is highest and the lotsize is lowest, Industry firms will locate at the city outskirts, because they need much space to produce, and have low transportation cost.

c) Agglomeration benefits: * Production gains by have all divisions located in the same center (such ass production, marketing, development etc.)
* Saved communication cost if the center is so large that simmular firms, suppliers and costumers are located there.

d) Assume profit maximizing firms, the larger the subcenter the larger the wages, due to increased commuting cost. This is a linear relationship.
Assume the larger the subcenter the smaller the agglomeration cost, such as saved communication cost. This is a non linear (kvasi-convex) relationship that runs perallell with the x-axis as size increases. Notation: C_i = Total cost = Wages (W) + Agglomeration ~~base~~ Cost (A).

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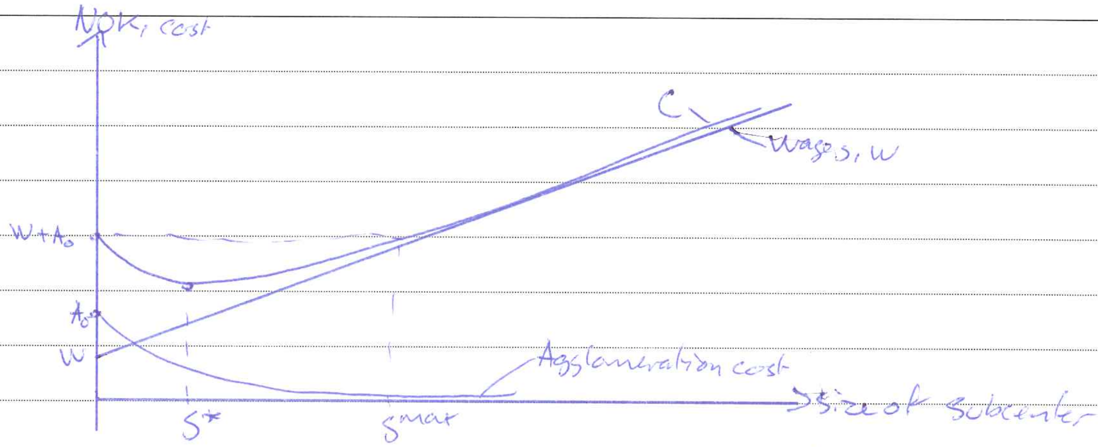
d)



Optimal size of the subcenter is where $\frac{\partial C}{\partial S}$ is equal zero. S^{max} is where it no longer is beneficial in adding one more firm in the subcenter. Agglomeration benefits let us have an optimal size of the subcenter, this is where the total cost are minimized, see illustration.

e) When a subcenter reaches $W + A_0$, then it is time to no longer lett other firms in to the subcenter, they should form a nother subcenter such that they get some agglomeration benefits, and hence ~~lower~~ higher profits. At S^{max}, there is no longer agglomeration benefits in the subcenter. So a subcenter should allways have a size between 0 and S^{max}, optimally a size of S^*.